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Contribution of sigma meson pole to K_L- K_S mass difference *

K. Terasaki
Yukawa Institute for Theoretical Physics,
Kyoto University, Kyoto 606-8502, Japan

Abstract

The hypothesis of σ meson pole dominance in the $|\Delta\mathbf{I}| = \frac{1}{2}$ $K \to \pi\pi$ amplitudes is tested qualitatively by using the K_L-K_S mass difference.

Dominance of σ-meson pole contribution in the amplitudes for the $K_S \to \pi\pi$ decays was first proposed as the origin of the well-known $|\Delta\mathbf{I}| = \frac{1}{2}$ rule in these decays [1], and recently revived in connection with the direct CP violation in the $K \to \pi\pi$ decays [2]. If it is the case, however, the matrix elements, $\langle\sigma|H_w|K\rangle$, should survive and give a significant contribution to the K_L-K_S mass difference, Δm_K, where H_w is the strangeness changing ($|\Delta S| = 1$) effective weak Hamiltonian.

Dynamical contributions of various hadron states to hadronic processes in which pion(s) take part can be estimated by using a hard pion technique (with PCAC) in the infinite momentum frame (IMF) [3]. For later convenience, we review briefly it below. As an example, we consider a decay, $B(p) \to \pi_1(q)\pi_2(p')$, in the IMF, i.e., $\mathbf{p} \to \infty$, and assume that its amplitude $M(B \to \pi_1\pi_2)$ can be approximately evaluated at a slightly unphysical point, $\mathbf{q} \to 0$, i.e., $q^2 \to 0$ but $(p \cdot q)$ is finite:

$$M(B \to \pi_1\pi_2) \simeq \lim_{\mathbf{p}\to\infty,\ \mathbf{q}\to 0} M(B \to \pi_1\pi_2). \tag{1}$$

In this approximation, the $\sigma \to \pi^+\pi^-$ amplitude is described in terms of the *asymptotic* matrix element, $\langle\pi^-|A_{\pi^-}|\sigma\rangle$, (matrix element of A_{π^-} taken between π^- and σ with infinite momentum) as

$$M(\sigma \to \pi^+\pi^-) \simeq \sqrt{2}\Big(\frac{m_\sigma^2 - m_\pi^2}{f_\pi}\Big)\langle\pi^-|A_{\pi^-}|\sigma\rangle, \tag{2}$$

which has been symmetrized with respect to exchange of π^+ and π^- in the final state since isospin symmetry is always assumed in this note. The asymptotic matrix element, $\langle\pi^-|A_{\pi^-}|\sigma\rangle$, is given by

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$$\lim_{\mathbf{p}\to\infty,\,\mathbf{q}\to 0}\langle\pi^-(p')|A_{\pi^-}|\sigma(p)\rangle$$
$$= (2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')\langle\pi^-|A_{\pi^-}|\sigma\rangle\sqrt{N_\pi N_\sigma}\Big|_{\mathbf{p}=\mathbf{p}'\to\infty} \quad (3)$$

and is related to the $\sigma\pi\pi$ coupling constant in the usual Feynman diagram approach [3], where N is the normalization factor of state vector.

Using the same technique, we can describe dynamical contributions of hadrons to the $K \to \pi\pi$ amplitude by a sum of equal-time commutator (ETC) term and surface term,

$$M(K\to\pi_1\pi_2) \simeq M_{\rm ETC}(K\to\pi_1\pi_2) + M_{\rm S}(K\to\pi_1\pi_2). \quad (4)$$

$M_{\rm ETC}$ has the same form as that in the old soft pion technique [4]

$$M_{\rm ETC}(K\to\pi_1\pi_2) = \frac{i}{\sqrt{2}f_\pi}\langle\pi_2|[V_{\bar{\pi}_1}, H_w]|K\rangle + (\pi_1\leftrightarrow\pi_2) \quad (5)$$

but it now should be evaluated in the IMF. The surface term,

$$M_S(K\to\pi_1\pi_2) = \lim_{\mathbf{p}\to\infty,\,\mathbf{q}\to 0}\Big\{-\frac{i}{\sqrt{2}f_\pi}q^\mu T_\mu\Big\} + (\pi_1\leftrightarrow\pi_2), \quad (6)$$

survives in contrast with the soft pion approximation and is now given by a sum of all possible pole amplitudes,

$$M_{\rm S} = \sum_n M_{\rm S}^{(n)} + \sum_l M_{\rm S}^{(l)}, \quad (7)$$

where the hypothetical amplitude T_μ has been given by

$$T_\mu = i\int e^{iqx}\langle\pi_2(p')|T[A_\mu^{(\bar{\pi}_1)}H_w]|K(p)\rangle d^4x. \quad (8)$$

$M_{\rm S}^{(n)}$ and $M_{\rm S}^{(l)}$ are pole amplitudes in the s- and u-channels, respectively, i.e.,

$$M_{\rm S}^{(n)}(K\to\pi_1\pi_2)$$
$$= \frac{i}{\sqrt{2}f_\pi}\Big(\frac{m_\pi^2-m_K^2}{m_n^2-m_K^2}\Big)\langle\pi_2|A_{\bar{\pi}_1}|n\rangle\langle n|H_w|K\rangle + (\pi_1\leftrightarrow\pi_2), \quad (9)$$
$$M_{\rm S}^{(l)}(K\to\pi_1\pi_2)$$
$$= \frac{i}{\sqrt{2}f_\pi}\Big(\frac{m_\pi^2-m_K^2}{m_l^2-m_\pi^2}\Big)\langle\pi_2|H_w|l\rangle\langle l|A_{\bar{\pi}_1}|K\rangle + (\pi_1\leftrightarrow\pi_2). \quad (10)$$

In this way, an approximate σ pole amplitude for the $K_S\to\pi^+\pi^-$ decay can be again described in terms of $\langle\pi^-|A_{\pi^-}|\sigma\rangle$ as

$$M^{(\sigma)}(K_S\to\pi^+\pi^-) \simeq i\frac{2}{f_\pi}\Big(\frac{m_\pi^2-m_K^2}{m_\sigma^2-m_K^2}\Big)\langle\pi^-|A_{\pi^-}|\sigma\rangle\langle\sigma|H_w|K^0\rangle. \quad (11)$$

Dominance of σ-meson pole in the $K_S\to\pi\pi$ amplitudes implies that $M^{(\sigma)}$ is much larger than the other contributions (the other pole amplitudes and $M_{\rm ETC}$ in addition to the factorized one, $M_{\rm fact}$, if it exists), i.e.,

$$|M^{(\sigma)}| \gg |M_{\rm ETC}|,\ |M_{\rm S}^{(n\neq\sigma)}|,\ |M_{\rm S}^{(l)}|,\ |M_{\rm fact}|, \tag{12}$$

unless the amplitudes in the right-hand-side cancel accidentally each other. However, if the σ pole contribution dominates $K_S \to \pi\pi$ amplitudes, it may be worried about that its strange partner, κ, also plays a role in the same amplitudes. The κ pole amplitude can be obtained in the same way as $M^{(\sigma)}$ and its ratio to $M^{(\sigma)}$ is approximately given by

$$\Big|\frac{M^{(\kappa)}(K_S \to \pi^+\pi^-)}{M^{(\sigma)}(K_S \to \pi^+\pi^-)}\Big| \sim \Big|\Big(\frac{m_\sigma^2 - m_K^2}{m_\kappa^2}\Big)\frac{\langle\pi|H_w|\kappa\rangle}{\langle\sigma|H_w|K\rangle}\Big|. \tag{13}$$

If $m_\kappa^2 > m_\sigma^2 \sim m_K^2$, the above ratio will be small unless $\langle\pi|H_w|\kappa\rangle$ is anomalously enhanced. However, if $m_\kappa^2 \sim m_\sigma^2 \gg m_K^2$, the κ pole can play a role in the $K_S \to \pi\pi$ amplitudes. Nevertheless, neglect of κ pole contribution does not change the essence of the physics in the K_L-K_S mass difference as will be seen later. Therefore, we will neglect the κ contribution to the $K_S \to \pi\pi$ amplitudes for simplicity.

The decay rates for $\sigma \to \pi^+\pi^-$ and $K_S \to \sigma \to \pi^+\pi^-$ are given by

$$\Gamma(\sigma \to \pi^+\pi^-) \simeq \frac{q_\sigma}{4\pi f_\pi^2 m_\sigma^2}(m_\sigma^2 - m_\pi^2)^2|\langle\pi^-|A_{\pi^-}|\sigma\rangle|^2, \tag{14}$$

and

$$\Gamma^{(\sigma)}(K_{\rm S} \to \pi^+\pi^-) \simeq \frac{q_K}{2\pi f_\pi^2 m_K^2}\Big(\frac{m_K^2 - m_\pi^2}{m_K^2 - m_\sigma^2}\Big)^2|\langle\pi^-|A_{\pi^-}|\sigma\rangle\langle\sigma|H_w|K^0\rangle|^2, \tag{15}$$

respectively, where q_σ and q_K are the center-of-mass momenta of the final pions in the corresponding decays. Since the $K_S \to \pi\pi$ mode dominates the decays of K_S, its total width, Γ_{K_S}, is approximately given by $\Gamma_{K_S} \simeq \frac{3}{2}\Gamma(K_S \to \pi^+\pi^-)$, so that $\Gamma_{K_S} \simeq \frac{3}{2}\Gamma^{(\sigma)}(K_S \to \pi^+\pi^-)$ under the σ pole dominance hypothesis.

The σ meson pole dominance in the $K_S \to \pi\pi$ means that the matrix element, $\langle\sigma|H_w|K\rangle$, exists and its magnitude should be sizable. Therefore, under this hypothesis, the σ meson pole may give a substantial contribution to Δm_K. The formula describing dynamical contributions of hadrons to Δm_K has been given in the IMF long time ago [5]. Using it, we obtain the following pole contribution of σ meson,

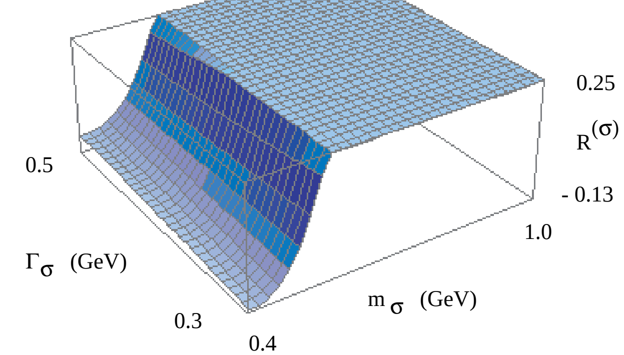
$$\Delta m_K^{(\sigma)} = -\frac{|\langle K_L|H_w|\sigma\rangle|^2}{2m_K(M_K^2 - m_\sigma^2)}, \tag{16}$$

where the matrix element, $\langle K_L|H_w|\sigma\rangle$, is again evaluated in the IMF. For later convenience, we consider the ratio of the K_L-K_S mass difference to the full width of K_S. If we assume the σ pole dominance in the $K_S \to \pi\pi$ decays, we obtain

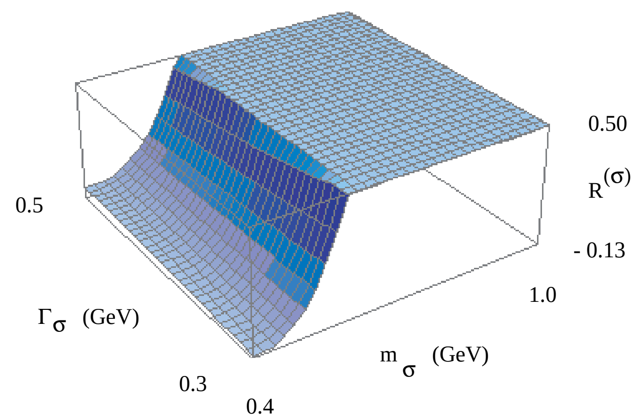
$$R^{(\sigma)} \equiv \frac{\Delta m_K^{(\sigma)}}{\Gamma_{K_S}} \simeq \frac{1}{2}\Big(\frac{q_\sigma}{q_K}\Big)\frac{(m_\sigma^2 - m_K^2)}{m_\sigma^2}\Big(\frac{m_\sigma^2 - m_\pi^2}{m_K^2 - m_\pi^2}\Big)^2\frac{m_K}{\Gamma_\sigma} \tag{17}$$

from Eqs. (14) – (16), where the full width of σ is given by $\Gamma_\sigma \simeq \frac{3}{2}\Gamma(\sigma \to \pi^+\pi^-)$ for m_σ less than the $K\bar{K}$ threshold ($\simeq$ 1 GeV).

Now we study whether the above σ pole dominance in the $K_S \to \pi\pi$ decays can be realized in consistency with Δm_K.



(a)



(b)

Fig. I. $R^{(\sigma)} = \Delta m^{(\sigma)}/\Gamma_{K_S}$ for $0.4 < m_\sigma < 1.0$ GeV and $0.3 < \Gamma_\sigma < 0.5$ GeV. $R^{(\sigma)}$ is cut at 0.25 in (a) and at 0.50 in (b) in order not to exceed the estimated $\pi\pi$ continuum contribution $R^{(\pi\pi)}$ and the measured $R_{\rm exp}$, respectively, as discussed in the text.

It has been known that contribution of S-wave $\pi\pi$ intermediate states to Δm_K can occupy about a half [6] of the observed value [7], i.e.,

$$R^{(\pi\pi)} \equiv \frac{\Delta m_K^{(\pi\pi)}}{\Gamma_{K_S}} = 0.22 \pm 0.03, \qquad R_{\rm exp} \equiv \frac{\Delta m_K}{\Gamma_{K_S}}\Big|_{\rm exp} = 0.477 \pm 0.022. \tag{18}$$

The above $\Delta m_K^{(\pi\pi)}$ was estimated by using the Muskhelishvili-Omnès equation and the measured $\pi\pi$ phase shifts, etc., in which any indication of σ meson was not obviously seen. Therefore, if σ exists, its contribution should be included in the above $\Delta m_K^{(\pi\pi)}$, so that we may put loosely the upper limit of the σ pole contribution to Δm_K around the above estimate of $\Delta m_K^{(\pi\pi)}$, i.e., $\Delta m_K^{(\sigma)}/\Gamma_{K_S} < 0.25$, and look for values of m_σ and Γ_σ to satisfy it since σ meson is still hypothetical, i.e., its mass and width are still not confirmed. At energies

lower than 900 MeV, the $\pi\pi$ phase shift analyses have excluded any narrow $I = 0$ scalar state but a broad one ($\Gamma_\sigma \sim 500$ MeV) may have a room in the region [7], $0.4 < m_\sigma < 1.2$ GeV. In fact, various broad candidates of σ meson with different masses ($\sim 500-700$ MeV), different widths ($\sim 300-600$ MeV) and different structures have been studied at this workshop [8].

$R^{(\sigma)}$ in Eq.(17) increases rapidly as m_σ increases. It is beyond not only the estimated $R^{(\pi\pi)}$ for $m_\sigma > 0.55$ GeV but also the measured $R_{\rm exp}$ in Eq.(18) for $m_\sigma > 0.57$ GeV and is much larger than the above cuts in the region $m_\sigma^2 \gg m_K^2$. Therefore, even if κ pole contribution to the $K \to \pi\pi$ decays is taken into account, the result, $R^{(\sigma)} \gg R_{\rm exp}$ for $m_\sigma^2 \gg m_K^2$, is not changed as discussed before. In this way, it is seen that the σ meson pole dominance in the $K \to \pi\pi$ amplitudes is not compatible with Δm_K if $m_\sigma > 0.57$ GeV and $0.3 < \Gamma_\sigma < 0.5$ GeV, unless any other contribution cancels $\Delta m_K^{(\sigma)}$.

However, the above does not necessarily imply that the σ meson pole dominance is compatible with the K_L-K_S mass difference if $m_\sigma < 0.55$ GeV, since we have so far considered only the long distance effects on the K_L-K_S mass difference. The short distance contribution from the box diagram [9] which is estimated by using the factorization may saturate the observed $(\Delta m_K)_{\rm exp}$ although it is still ambiguous because of uncertainty of the so-called B_K parameter. If it is the case, however, we need some other contribution to cancel the $\pi\pi$ continuum contribution (including σ meson pole). Possible candidates are pseudo-scalar(PS)-meson poles since the other contributions of multi hadron intermediate states will be small because of their small phase space volumes. The above implies that the matrix elements, $\langle P|H_w|K\rangle$, $P = \pi^0, \eta, \eta', \cdots$, survive and their sizes are large enough to cancel $\Delta m_K^{(\pi\pi)}$. In this case, however, $\langle \pi|H_w|K\rangle$'s can give large effects on the $K \to \pi\pi$ amplitudes [10] through Eq.(4) with Eq.(5) and break the σ meson pole dominance.

For the $K_L \to \gamma\gamma$ decay, it is known that short distance contribution is small [9]. To reproduce the observed rate for this decay, we again need contributions of PS-meson poles given by the matrix elements, $\langle P|H_w|K\rangle$'s, with sufficient magnitude, although their contributions are sensitive to the η-η' mixing and are not always sufficient. In fact, the above PS-meson matrix elements can approximately reproduce $\Gamma(K_L \to \gamma\gamma)_{\rm exp}$, $\Gamma(K \to \pi\pi)_{\rm exp}$'s and $(\Delta m_K)_{\rm exp}$, simultaneously, with the help of some other contributions (non-factorizable amplitudes with PS- and K^*-meson poles for the $K_L \to \gamma\gamma$ decay, factorized ones for the $K \to \pi\pi$ decays and the short distance contribution to the K^0-$\bar{K}^0$ mixing, etc.) but without any contribution of σ pole [10]. Namely, we do not necessarily need the σ pole contribution in the $K_S \to \pi\pi$ decays.

As was seen above, it is unlikely that the σ meson pole amplitude dominates the $K_S \to \pi\pi$. It will be seen directly by comparing $M^{(\sigma)}(K_S \to \pi^+\pi^-)$ with $M_{\rm ETC}(K_S \to \pi^+\pi^-)$. If the asymptotic matrix elements, $\langle \pi|H_w|K\rangle$'s, with sufficient magnitude exist and satisfy the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule (as derived by using a simple quark model [10] or as required to realize the same rule in the $K \to \pi\pi$ amplitudes, i.e., $M_{\rm ETC}(K^+ \to \pi^+\pi^0) = 0$), we obtain

$$\Big|\frac{M^{(\sigma)}(K_S^0 \to \pi^+\pi^-)}{M_{\rm ETC}(K_S^0 \to \pi^+\pi^-)}\Big| \simeq 2\Big|\Big(\frac{m_K^2 - m_\pi^2}{m_\sigma^2 - m_K^2}\Big)\langle \pi^-|A_{\pi^-}|\sigma\rangle \frac{\langle \sigma|H_w|K^0\rangle}{\langle \pi^+|H_w|K^+\rangle}\Big|. \tag{19}$$

The mass dependent factor $|(m_K^2 - m_\pi^2)/(m_\sigma^2 - m_K^2)|$ from $M^{(\sigma)}$ can be enhanced only if m_σ is very close to m_K and σ is narrow. However, if Γ_σ were small, $|\langle \pi^-|A_{\pi^-}|\sigma\rangle|$ also would be small. When we smear out the singularity at $m_\sigma = m_K$ using the Breit-Wigner form, the size of $|(m_K^2 - m_\pi^2)/(m_\sigma^2 - m_K^2)\langle \pi^-|A_{\pi^-}|\sigma\rangle|$ is at most $\simeq 2$ for $0.4 < m_\sigma < 1.0$ GeV and

$0.3 < \Gamma_\sigma < 0.5$ GeV. However, any narrow σ state around m_K is not allowed [7] as mentioned before. Moreover, σ does not belong to the same ground state as π and K (for example, ${}^3P_0$ of $\{q\bar{q}\}$ state in the quark model, etc.), so that the matrix elements, $|\langle\sigma|H_w|K\rangle|$, will be much smaller than $|\langle\pi|H_w|K\rangle|$ since wave function overlapping between σ and K meson states will be much smaller than that between π and K which belong to the same ${}^1S_0$ state of $\{q\bar{q}\}$. Therefore, it is unlikely that the σ meson pole amplitude dominates the $K \to \pi\pi$ amplitudes.

An amplitude for dynamical hadronic process can be decomposed into (*continuum contribution*) + (*Born term*). Since $M_{\rm S}$ has been given by a sum of pole amplitudes, $M_{\rm ETC}$ corresponds to the continuum contribution [11]. In the present case, $M_{\rm ETC}(K_S \to \pi\pi)$ will be dominated by contributions of isoscalar S-wave $\pi\pi$ intermediate states and develop a phase ($\simeq$ isoscalar S-wave $\pi\pi$ phase shift at m_K) relative to the Born term which is usually taken to be real in the narrow width limit. The estimated phase difference between $|\Delta\mathbf{I}| = \frac{1}{2}$ and $\frac{3}{2}$ amplitudes for the $K \to \pi\pi$ decays is close to the measured isoscalar S-wave $\pi\pi$ phase shift at m_K [12]. It suggests that the isoscalar S-wave $\pi\pi$ continuum contribution will be dominant in the $K_S \to \pi\pi$ amplitudes.

In summary, we have studied contribution of the σ meson pole to Δm_K under the hypothesis that σ meson pole dominates the $K_S \to \pi\pi$ amplitudes, and have seen that it provides too large contributions to Δm_K and that, to cancel out such effects, contributions of pseudo-scalar-meson poles will be needed. We also have discussed, comparing the σ meson pole amplitude with $M_{\rm ETC}$ in the $K_S \to \pi\pi$ amplitudes, that enhancement of the σ meson pole contribution is not sufficient if it is broad. Additionally, a recent analysis in the $K \to \pi\pi$ decays within the theoretical framework of non-linear σ model suggests that the σ meson pole contribution can occupy, at most, about a half of the $|\Delta\mathbf{I}| = \frac{1}{2}$ amplitude [13]. Therefore, we conclude that the σ pole dominance in the $|\Delta\mathbf{I}| = \frac{1}{2}$ amplitude for the $K \to \pi\pi$ decays is very unlikely.

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REFERENCES

[1] S. Oneda, private communication (1983).
E. P. Shabalin, Sov. J. Nucl. Phys. **48**, 172 (1988).
T. Morozumi, C. S. Lim and A. I. Sanda, Phys. Rev. Lett. **65**, 404 (1990).

[2] Y. -Y. Keum, U. Nierste and A. I. Sanda, Phys. Lett. **B457**, 157 (1999).

[3] K. Terasaki, S. Oneda, and T. Tanuma, Phys. Rev. D **29**, 456 (1984).
S. Oneda and K. Terasaki, Prog. Theor. Phys. Suppl. **82**, 1 (1985) and references therein.

[4] H. Sugawara, Phys. Rev. Lett. **15**, 870 and 997 (E) (1965).
M. Suzuki, Phys. Rev. Lett. **15**, 986 (1965).

[5] C. Itzykson, M. Jacob and G. Mahoux, Suppl. Nuovo Cim. **V**, 978 (1967).

[6] M. R. Pennington, Phys. Lett. B **153**, 439 (1985).

[7] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. **3**, 1 (1998) and references therein.

[8] N. A. Tornqvist, the summary talk at this workshop and references therein.

[9] M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).

[10] K. Terasaki, Report No. YITP-99-42, hep-ph/9908428.

[11] V. S. Mathur and L. K. Pandit, *Advances in Particle Physics*, edited by R. L. Cool and R. E. Marshak (Interscience Publishers, New York, 1968), Vol. 2, p. 383.

[12] A. N. Kamal, J. Phys. G: Nucl.Phys. **12**, L43 (1986) and references quoted therein.

[13] M. Harada, Y. Keum, Y. Kiyo, T. Morozumi, T. Onogi and N. Yamada, Report No. HUPD-9913, KEK-TH-643, DPNU-99-27, hep-ph/9910201.